Problem Sheet 2b - Probability Mass Distributions

## Question 1

1. There are 30 candy covered chocolates in a bag M&M’s. There is a .1 probability that that the candy is red. If X is the number of red M&M’s in the bag.

i Give the binomial probability mass function for X.

**ANSWER:**

where k is the number of red M & Ms.

ii Find the probability of less than 2 red M&Ms in the bag.

**ANSWER:**

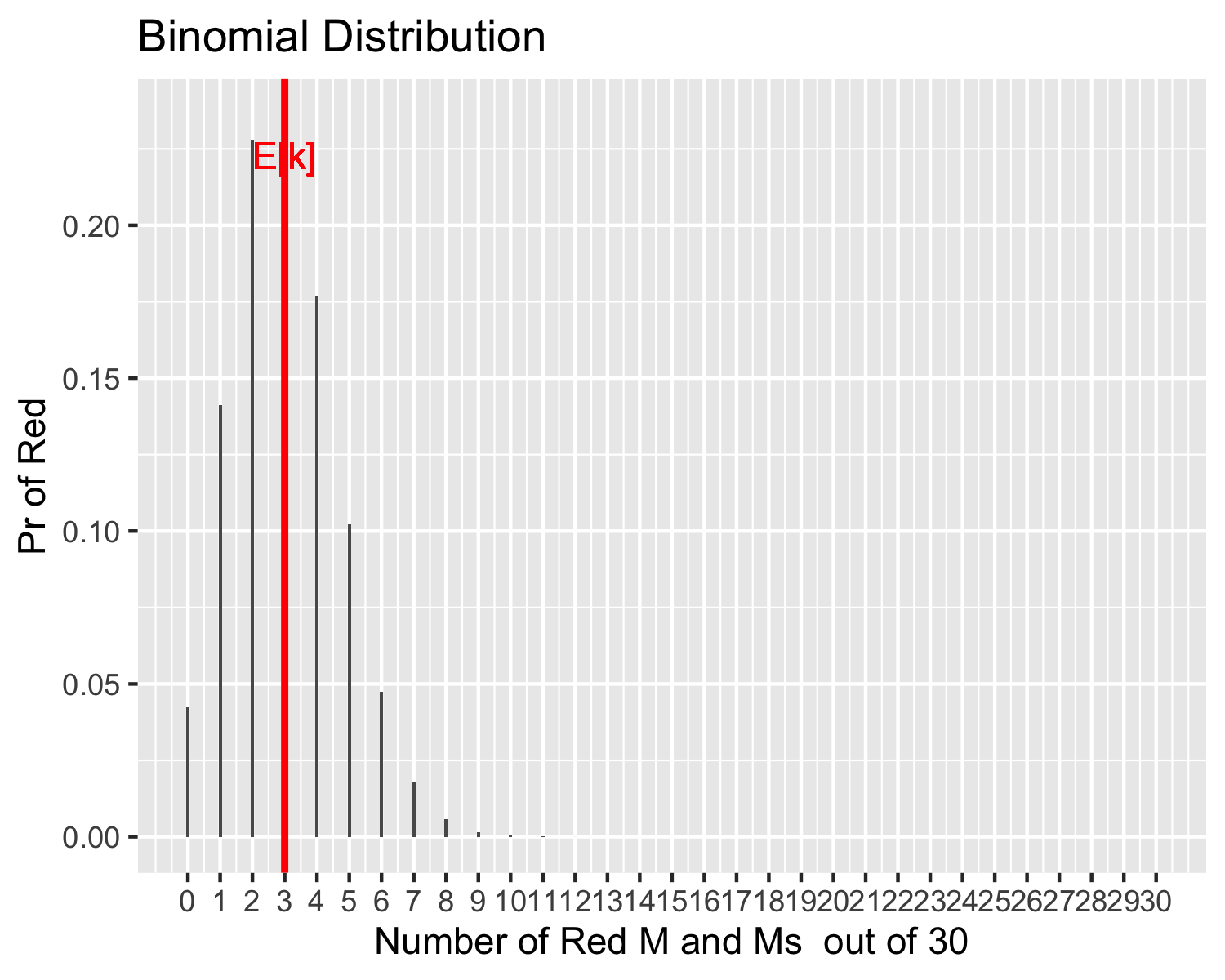
n<-30  
k<-1  
MandM<-0:n # Number of Games  
Pr\_Red<-0.1 # Probability of Success  
Pr\_Reds<-c(dbinom(0:n,n,Pr\_Red)) # Binomial Probability   
Q1ii<-c(dbinom(0:k,n,Pr\_Red))  
E\_X<- n\*Pr\_Red # Expected Value  
Var\_X<-10\*Pr\_Red\*(1-Pr\_Red) # Variance

As it is a Binomial Distribution we can state that the expected number of Red is

the variance of the distribution is

The plot below shows the Binomial Distribution M & Ms:

df <- data.frame(MandM,Pr\_Reds)  
## Plot  
binomial.p\_dist<-ggplot(df, aes(x=MandM,y=Pr\_Reds)) + geom\_col(width = 0.1)+  
 xlab("Number of Red M and Ms out of 30")+  
 ylab("Pr of Red")+ggtitle("Binomial Distribution")+  
 scale\_x\_continuous(breaks=0:n)+   
 geom\_vline(xintercept = E\_X, color = "Red", size=1)+  
 geom\_text(aes(x=E\_X, label="E[k]", y=max(Pr\_Reds)),vjust=2,   
 colour="red", text=element\_text(size=18))  
  
 binomial.p\_dist



ggsave("Q1.png",dpi=300, width = 4, height = 2)

## Question 2

1. A baby wakes on average 0.25 times every hour.

i If X is the number of times a baby wakes in an hour, give the poisson probability mass function for X. **ANSWER:**

The distribution is described by the average, ,

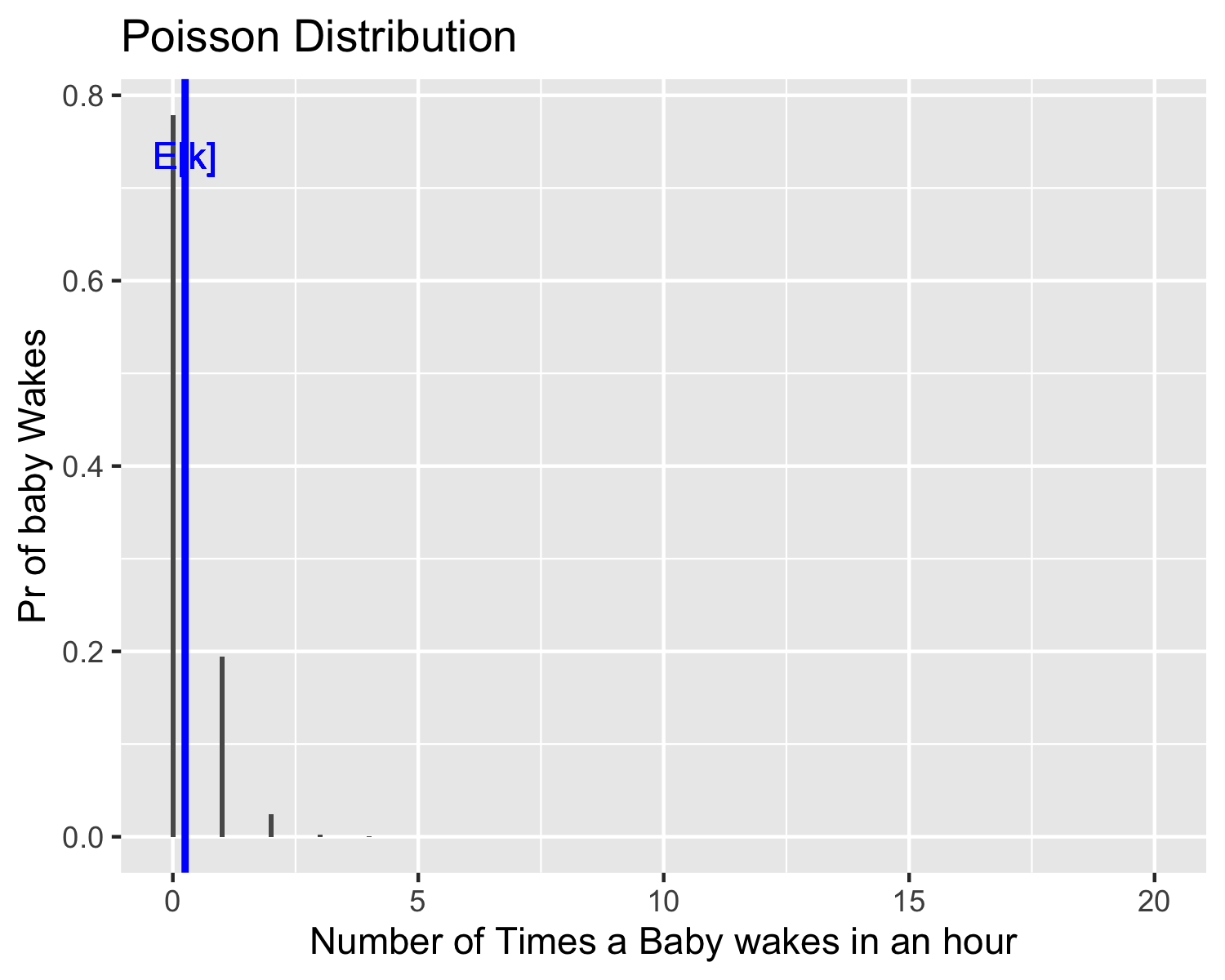
where is the number of times the baby wakes every hour.

The expected value of the Poisson Distribution is

with the variance

The plot below shows the Poisson Distribution for average number of times a baby wakes in an hour:

Wake<-0:20 # Number of Games  
Baby\_wakes<-0.25 # Lambda  
Pr\_wakes<-c(dpois(Wake,Baby\_wakes)) # Poisson Probabilities  
E\_X<-Baby\_wakes # Expected Outcome  
  
df <- data.frame(Wake,Pr\_wakes)  
  
## Plot  
Poisson.p\_dist<-ggplot(df, aes(x=Wake,y=Pr\_wakes)) +   
 geom\_col(width = 0.1)+xlab("Number of Times a Baby wakes in an hour")+  
 ylab("Pr of baby Wakes")+ggtitle("Poisson Distribution")+   
 geom\_vline(xintercept = E\_X, color = "blue", size=1)+  
 geom\_text(aes(x=E\_X, label="E[k]", y=max(Pr\_wakes)),vjust=2,  
 colour="blue",text=element\_text(size=18))  
  
Poisson.p\_dist



ggsave("Q2i.png",dpi=300, width = 4, height = 2)

ii If X is the number of times a baby wakes in eight hour, give the poisson probability mass function for X.

**ANSWER:**

The distribution is described by the average, ,

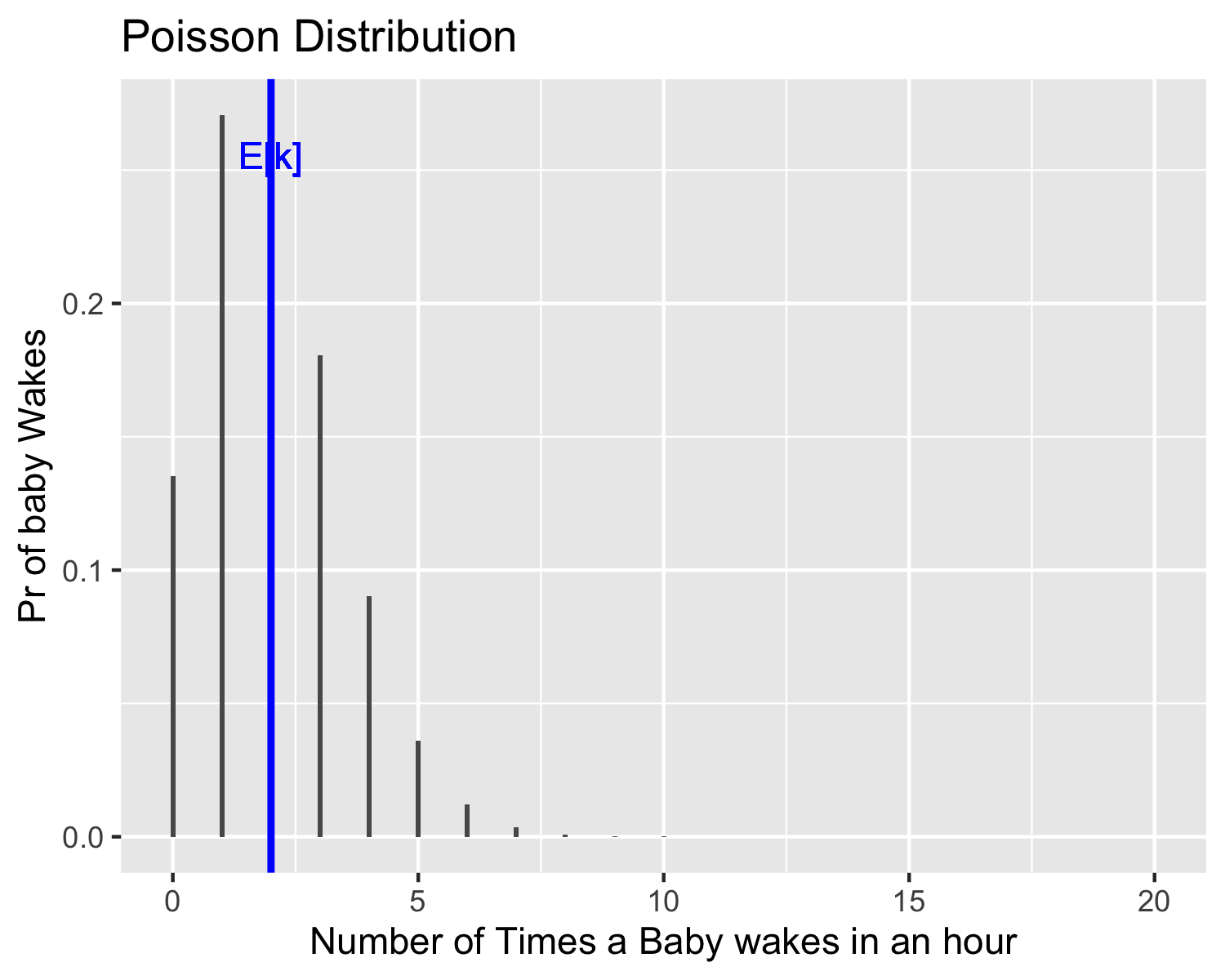
where is the number of times the baby wakes every 8 hours.

The expected value of the Poisson Distribution is

with the variance

The plot below shows the Poisson Distribution for average number of times a baby wakes in eight hours:

Baby\_wakes<-0.25\*8 # Lambda  
Pr\_wakes<-c(dpois(Wake,Baby\_wakes)) # Poisson Probabilities  
E\_X<-Baby\_wakes # Expected Outcome  
  
df <- data.frame(Wake,Pr\_wakes)  
  
## Plot  
Poisson.p\_dist<-ggplot(df, aes(x=Wake,y=Pr\_wakes)) +   
 geom\_col(width = 0.1)+xlab("Number of Times a Baby wakes in an hour")+  
 ylab("Pr of baby Wakes")+ggtitle("Poisson Distribution")+   
 geom\_vline(xintercept = E\_X, color = "blue", size=1)+  
 geom\_text(aes(x=E\_X, label="E[k]", y=max(Pr\_wakes)),vjust=2,  
 colour="blue",text=element\_text(size=18))  
  
Poisson.p\_dist



ggsave("Q2i.png",dpi=300, width = 4, height = 2)

iii What is the probability that the baby does not wake during the 8 hours.

**ANSWER:**

Baby\_wakes<-0.25\*8 # Lambda  
Pr\_wakes\_zero<-dpois(0,Baby\_wakes) #

## Question 3

1. Give the features of a

i Geometric Experiment.

**ANSWER:**

• Bernoulli Trial • Goes until you “win” • E[X]=1/p, VAR[X]=q/p^2

ii Binomial Experiment.

**ANSWER:**

• Bernoulli Trial • Play a specific number of times • E[x]=np, VAR[X]=npq

iii Poisson Experiment.

**ANSWER:**

• Well known mean • Number of “wins” • E[X]=lambda, VAR[X]=lambda

## Question 4

1. Every day a production line makes 100 computers of which 10% are defective. If X is the number of defective computers in a day.
2. Give the binomial probability mass function for X.

**ANSWER:**

where k is the number of defective computers.

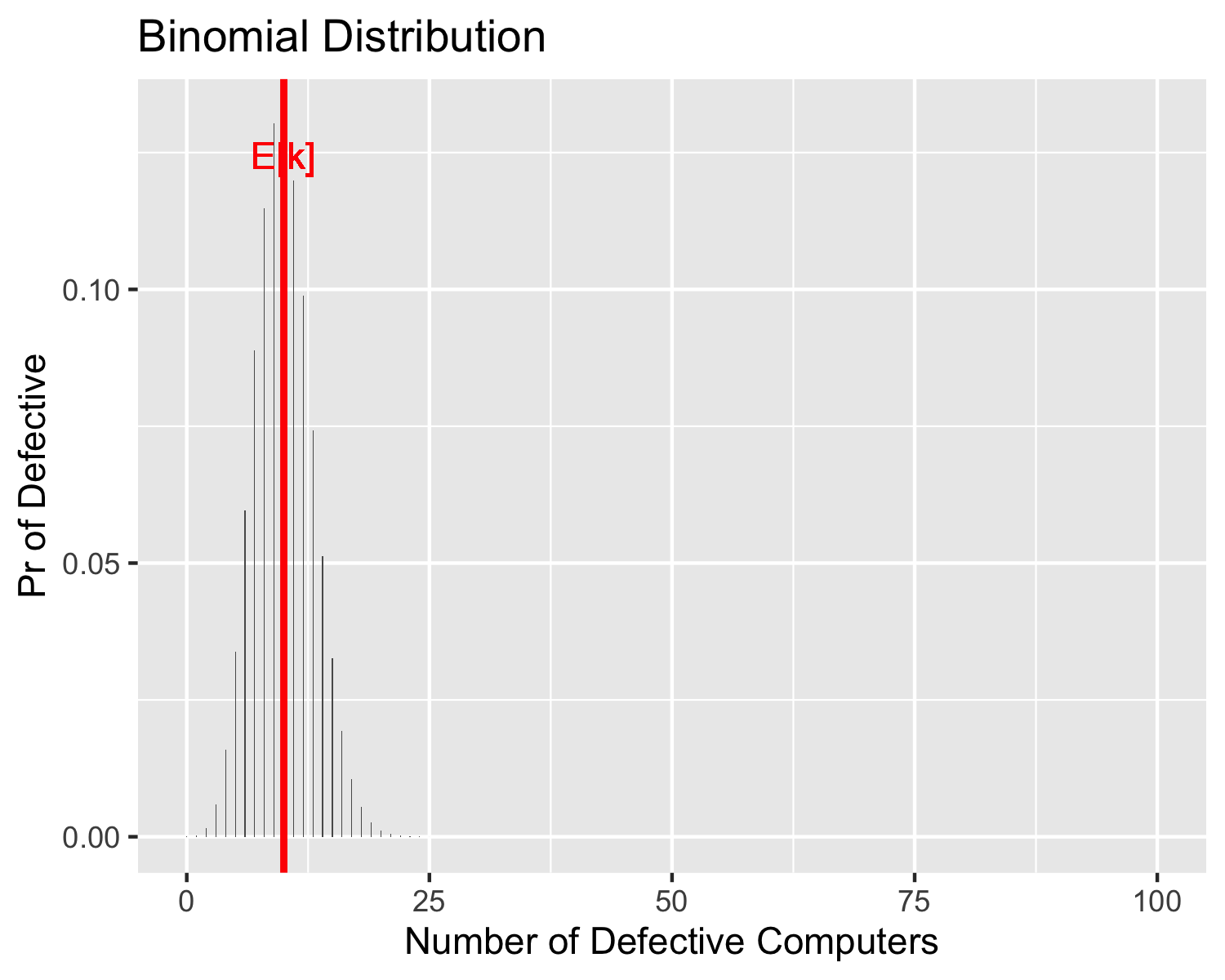
1. Find the probability that there is more than 2 computers defective in a day.

**ANSWER:**

n<-100  
k<-2  
Computers<-0:n # Number of Games  
Pr\_Def<-0.1 # Probability of Success  
Pr\_Defs<-c(dbinom(0:n,n,Pr\_Def)) # Binomial Probability   
Q4ii<-c(dbinom(0:k,n,Pr\_Def))  
E\_X<- n\*Pr\_Def # Expected Value  
Var\_X<-10\*Pr\_Def\*(1-Pr\_Def) # Variance

The plot below shows the Binomial Distribution for defective computers:

df <- data.frame(Computers,Pr\_Defs)  
## Plot  
binomial.p\_dist<-ggplot(df, aes(x=Computers,y=Pr\_Defs)) + geom\_col(width = 0.1)+  
 xlab("Number of Defective Computers")+  
 ylab("Pr of Defective")+ggtitle("Binomial Distribution")+  
 geom\_vline(xintercept = E\_X, color = "Red", size=1)+  
 geom\_text(aes(x=E\_X, label="E[k]", y=max(Pr\_Defs)),vjust=2,   
 colour="red", text=element\_text(size=18))  
  
 binomial.p\_dist



ggsave("Q4.png",dpi=300, width = 4, height = 2)

## Question 5

1. A phone center receives 15 calls every 30 minutes.
2. If X is the number of phone calls in 30 minutes, give the Poisson probability mass function for X.

**ANSWER:**

The distribution is described by the average, ,

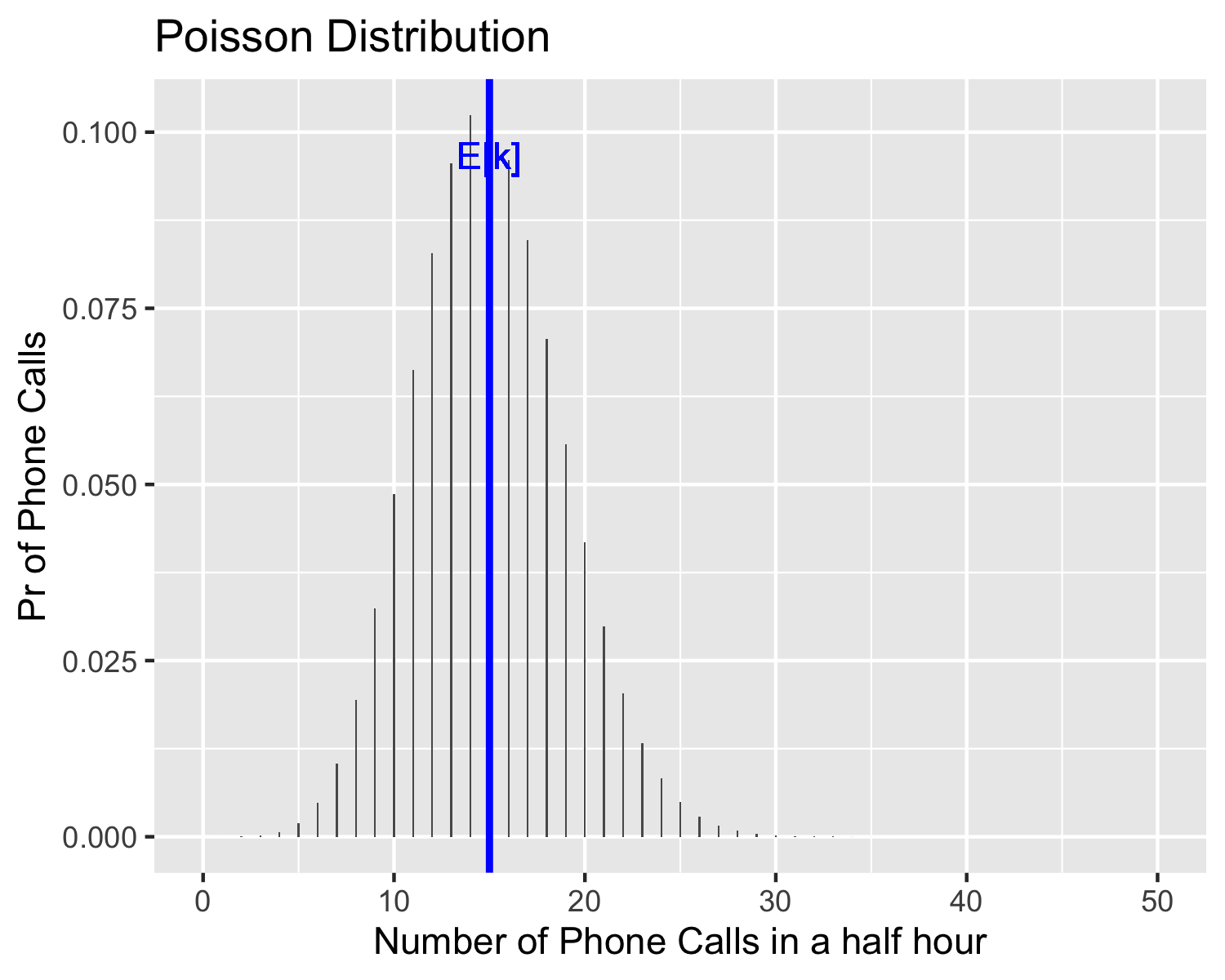
where is the number of phone calls per half hour.

The expected value of the Poisson Distribution is

with the variance

The plot below shows the Poisson Distribution for average number of calls per half-hour:

Calls<-0:50 # Number of Games  
No\_of\_Calls<-15 # Lambda  
Pr\_Calls<-c(dpois(Calls,No\_of\_Calls)) # Poisson Probabilities  
E\_X<-No\_of\_Calls # Expected Outcome  
  
df <- data.frame(Calls,Pr\_Calls)  
  
## Plot  
Poisson.p\_dist<-ggplot(df, aes(x=Calls,y=Pr\_Calls)) +   
 geom\_col(width = 0.1)+xlab("Number of Phone Calls in a half hour")+  
 ylab("Pr of Phone Calls")+ggtitle("Poisson Distribution")+   
 geom\_vline(xintercept = E\_X, color = "blue", size=1)+  
 geom\_text(aes(x=E\_X, label="E[k]", y=max(Pr\_Calls)),vjust=2,  
 colour="blue",text=element\_text(size=18))  
  
Poisson.p\_dist



ggsave("Q5i.png",dpi=300, width = 4, height = 2)

1. What is the probability that there will be exactly 10 phone calls in the first 30 minutes and exactly 20 phone calls in the second 30 minutes.

**ANSWER:**

No\_of\_Calls<-15 # Lambda  
Pr\_Calls\_10<-dpois(10,No\_of\_Calls) # 10 Calls Poisson Probability  
Pr\_Calls\_20<-dpois(20,No\_of\_Calls) # 20 Calls Poisson Probability  
Pr\_Calls\_10\_then\_20 <- Pr\_Calls\_10\*Pr\_Calls\_20

Probability of 10 calls in the first half-hour:

Probability of 20 calls in the first half-hour:

The combination of both: